

Seismology of Rapidly Rotating Accreting White Dwarfs

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Abstract

A number of White Dwarfs (WDs) in cataclysmic binaries have shown brightness variations consistent with non-radial oscillations as observed in isolated WDs. A few objects have been well-characterized with photometric campaigns in the hopes of gleaning information about the mass, spin, and possibly internal structural characteristics. The novel aspect of this work is the possibility to measure or constrain the interior structure and spin rate of WDs which have spent gigayears accreting material from their companion, undergoing thousands of nova outbursts in the process. In addition, variations in the surface temperature affect the site of mode driving, and provide unique and challenging tests for mode driving theories previously applied to isolated WD's. Having undergone long-term accretion, these WDs are expected to have been spun up. Spin periods in the range 60-100 seconds have been measured by other means for two objects, GW Lib and V455 And. Compared to typical mode frequencies, the spin frequency may be similar or higher, and the Coriolis force can no longer be treated as a small perturbation on the fluid motions. We present the results of a non-perturbative calculation of the normal modes of these WDs, using interior thermal structures appropriate to accreting systems. This includes a discussion of the surface brightness distributions, which are strongly modified from the non-rotating case. Using the measured spin period of approximately 100 seconds, we show that the observed pulsations from GW Lib are consistent with the three lowest azimuthal order rotationally modified modes that have the highest frequency in the stellar frame. The high frequencies are needed for the convective driving, but are then apparently shifted to lower frequencies by a combination of their pattern motion and the WD rotation.

1. Introduction

There are now more than a dozen systems known in which the accreting WD exhibits pulsational variations in its brightness consistent with non-radial oscillations as observed in isolated WDs [1]. The first discovered and best studied example is GW Lib [2]. We performed an analysis of GW Lib under the assumption of slow rotation [3], but pointed out at the time that these WDs, having spent gigayears accreting matter via an accretion disk, are likely to have been spun up. The framework for treating spin in these cases was developed in the context of ocean waves on neutron stars [4], and with certain caveats can be extended to uniformly rotating WDs. With the advent of a measured spin and mass for GW Lib [5], enough parameters are known about this object that we can construct a consistent picture of the identity of its observed pulsations. A critical check is whether or not these appear consistent with being driven by their interaction with the surface convection zone [6].

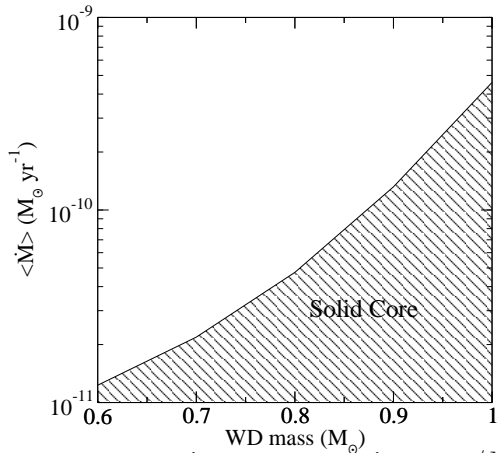


Fig. 1. Region of parameter space in average accretion rate, $\langle \dot{M} \rangle$, and WD mass in which the center of the WD is expected to be solid (crystalline). This is determined by the equilibrium central temperature.

2. White Dwarf Thermal State

A critical ingredient for modelling the pulsation due to global normal modes is the internal thermal state of the accreting WD, which is quite different than an isolated WD [7]. Whereas an isolated WD’s surface temperature/flux, T_{eff} , is directly related to its core temperature, T_c (see e.g. [8]), in an accreting system, T_{eff} is determined principally by the mass transfer rate onto the WD and its overall mass, M_{WD} [9]. Thus measurement of T_{eff} provides only weak constraints on T_c , and a thermal structure of even the WD envelope from isolated systems is entirely inappropriate. Accreting WDs with T_{eff} near the instability strip have significantly lower T_c than isolated WDs at similar T_{eff} .

The WD is “re-heated” by the accretion up to an equilibrium T_c determined by the balance of heat input by accretion from compression of the accreted material (*not* from infall energy) and a low level of nuclear fusion leading up to the nova runaway, and loss of energy via observed radiation between nova outbursts [7]. The speed that the re-heating takes place is also dependent on the accretion rate, being faster for higher $\langle \dot{M} \rangle$. Above the period gap, where $\langle \dot{M} \rangle \sim 10^{-9} M_{\odot} \text{ yr}^{-1}$, there is not enough time for the WD to reach equilibrium. However, below the period gap, the $\langle \dot{M} \rangle \sim 10^{-11} M_{\odot} \text{ yr}^{-1}$ is nearly constant for approximately 4 Gyrs [10], which is more than enough time for the WD to come to equilibrium, even at such a low $\langle \dot{M} \rangle$.

As an example of the critical information garnered from the thermal WD state is shown in Figure 1., where the presence or absence of a solid core is indicated as a function of the M_{WD} and $\langle \dot{M} \rangle$. Here we assume a carbon-oxygen core. Note that, given the typical $\langle \dot{M} \rangle$ of a few $\times 10^{-11} M_{\odot} \text{ yr}^{-1}$, accreting pulsating WDs have solid cores at much lower masses than their isolated pulsating counterparts [11]. This is due to the surface luminosity being provided by compression of surface material due to accretion rather than cooling of the core, leading to a low T_c as mentioned above.

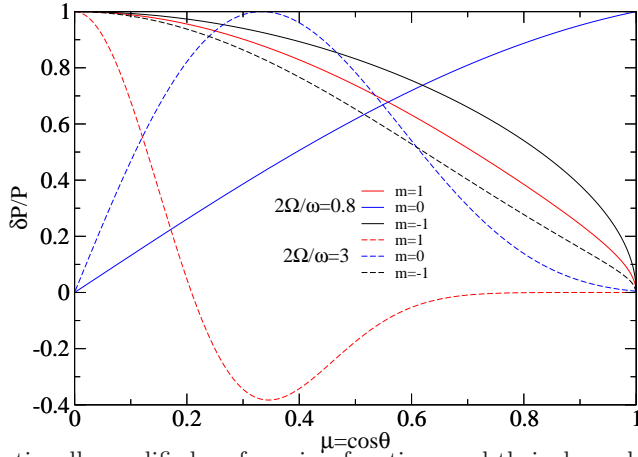


Fig. 2. Rotationally modified surface eigenfunctions and their dependence on the ratio of spin to mode frequency $q = 2\Omega/\omega$.

3. Seismology Under Rapid Rotation and Driving

When the spin frequency becomes similar or larger than the mode frequency, rotation cannot be treated in a perturbative fashion. The spin breaks the spherical symmetry of the problem and introduces a Coriolis force into the equations for fluid motion. Methods for treating this case were adapted from Geophysics for Neutron star oceans by [4], and we will make use of this method here. The approximation is called the “traditional approximation” and depends upon the assumption that $N^2 \gg \Omega\omega k_r/k_t$ and $k_r \gg k_t$ where N is the Brunt-Vaisala (buoyancy) frequency, Ω is the stellar spin frequency, ω is the mode frequency, and k_r and k_t are the radial and tangential wave numbers respectively. These conditions are satisfied in the WD except near the center, where N is small. However, for the models studied here, the center is solid, so that waves do not propagate there.

The traditional approximation results in a separation of the angular from the radial equations for the perturbed fluid motion equations. The differential equation for the angular part, which previously had spherical harmonic solutions, is replaced by Laplace’s tidal equation [4]. Solutions to this equation depend on the parameter $q = 2\Omega/\omega$ in addition to ℓ and m . The radial differential equation for the mode shape (namely giving $\xi_r(r)$ and $\xi_t(r)$, the radial and tangential motions) recovers a form just like the non-rotating case except that $\ell(\ell+1)$ is replaced by an eigenvalue λ determined by Laplace’s tidal equation and depending on q , ℓ and m .

The action of the Coriolis force is to tend to “squeeze” modes towards the equator. This can be seen from Figure 2., which shows the form of several of the angular eigenfunctions in $\mu = \cos\theta$ for two different values of q , where θ is the classical polar coordinate so that $\mu = 0$ is the equator and $\mu = 1$ is the pole. This function also determines the visibility of perturbations on the surface, so that the strongest perturbations for the $m = 1$ modes, for example, will be concentrated near the equator and this will become more so at higher q . Higher q corresponds to either higher Ω or lower ω (at fixed Ω) and therefore higher radial order g-modes.

On WDs, g-mode oscillations are known to be driven by the interaction of the modes with the surface convection zone [12]. This feedback process drives modes whose frequency is greater than approximately the thermal time at the base of the

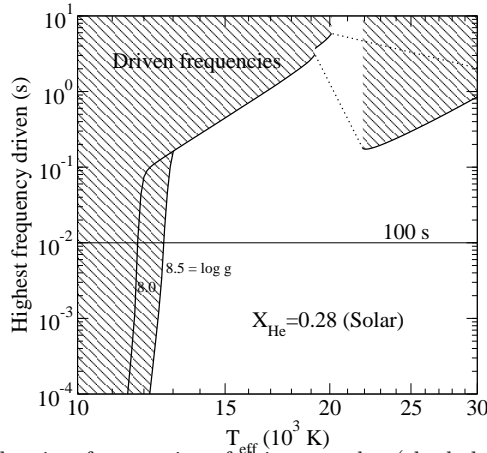


Fig. 3. Diagram showing frequencies of driven modes (shaded region) for a range of WD effective temperatures.

convection zone. Since the highest frequency g-modes in a WD have frequencies of around 0.005 Hz or so, they are not excited above $T_{\text{eff}} \approx 12 - 13$ kK. There are also important effects of both the abundance of the accreted material (and therefore the convection zone) and the WD mass [6]. Figure 3. shows roughly the frequencies expected to be driven for various T_{eff} values. Also indicated is frequency corresponding to a mode with a 100 second period.

4. Application to GW Lib

In the case of GW Lib, [5] measured the spin period from the width of absorption lines to be approximately 100 s. This implies that the g-modes will be strongly modified by rotation. The T_{eff} of GW Lib has been measured [13] from UV spectroscopy to be 13-16 kK, but is subject to some ambiguity due to a single-temperature fit not reproducing certain features of the spectrum. We carried out a calculation of the rotationally-modified g-modes for an object similar to our previous study of GW Lib [3] with $M = 0.9M_{\odot}$ and an accreted layer mass of half the nova ignition mass. The mode frequencies in the star's frame (the fluid frame) for $\ell = 1$ are shown in the left panel of Figure 4. as a function of the stellar spin. The modes are split as expected according to their m values at low spin, but then become strongly shifted at higher spins, overlapping modes of other m values. This features makes identifying mode structure with no knowledge of the possible WD spin challenging.

Also indicated in the left panel is a possible limit on the driven mode frequencies which would cause only the 4 $m = 1$ g-modes with the highest frequencies to be driven for $f_{\text{spin}} = 0.01$ Hz. Modes with higher ℓ values may also be driven, but, as in isolated WDs, would not show up as strongly as surface perturbations due to cancellation. These modes frequencies are highlighted with heavy lines in both the left and right panels of Figure 4..

Both the $m = -1$ and $m = +1$ modes form a brightness pattern which propagates with respect to the surface of the WD, in pro- and retrograde directions respectively. When combined with the spin, this causes a fixed observer to see a different frequency of oscillation than that of the oscillation on the WD. The observed frequency is $f_{\text{obs}} = |f_{\text{mode}} - mf_{\text{star}}|$. The $m = 0$ mode has the same frequency on the star and to an observer because it has no rotating pattern. Taking the $m = 1$ modes as an example, at low spin the retrograde propagation of

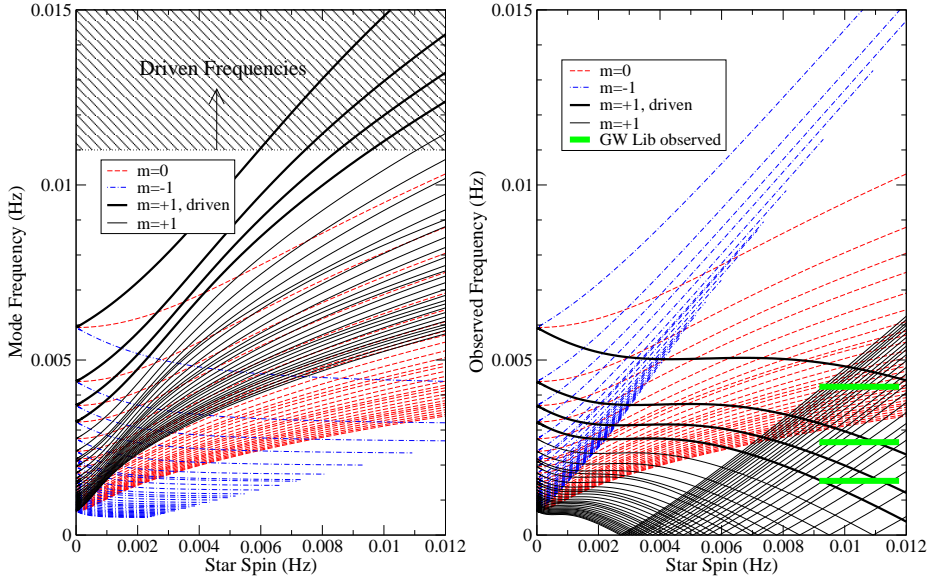


Fig. 4. Rotationally modified g-mode frequencies as a function of spin for $\ell = 1$. The left panel shows the frequencies in the (rotating) star’s frame and the right panel shows the frequency observed by a stationary (non-rotating) observer. Also indicated in the star frame is a driving range which would excite only the highest 4 modes at a stellar spin of 0.01 Hz. These modes are highlighted in both figures. Finally the frequencies observed in GW Lib are indicated with the observed spin frequency and its uncertainty.

the modes with $\omega \gg \Omega$ causes the observed frequency to be near the non-rotating g-mode frequency, but slightly lower. As the spin becomes relatively stronger, the mode is “dragged” along with the star, until it eventually passes through being stationary (zero frequency) and becomes prograde as seen by the observer.

The three thick horizontal lines in the bottom right of the right panel of Figure 4. indicate the observed frequencies of pulsations on GW Lib. The horizontal extent of these lines corresponds to the uncertainty in the spin given by [5]. Note that the present WD model has *not* been fit to these lines. The relative spacing among these lines is sensitive mainly to a combination of the $\langle \dot{M} \rangle$, the accreted layer mass, and M_{WD} . The near consistency without a fit indicates that there is some combination of these that will allow the simultaneous fit of these periods at a spin frequency that is consistent with the observations. It is also consistent with the mode driving that only these three modes are driven, since they are the highest frequency modes in the star frame. Additionally, higher radial order modes, which have a smaller ω , will be more tightly constrained to the equator, possibly limiting their visibility.

5. Conclusion

Motivated by the recent measurement of the spin of the accreting WD GW Lib, we have presented a consistent picture for the pulsations observed. In this picture the 3 independent pulsations observed are due to the 3 highest frequency $\ell = 1$

rotationally modified g-modes. These modes have $m = 1$ and have a retrograde pattern propagation so that to a fixed observer they have an apparently lower frequency than on the star. The higher frequency of these modes in the star's frame (above 0.01 Hz) also allows GW Lib to enter the instability strip at higher T_{eff} than it would have if it were not rotating rapidly.

While here we have shown the structure of $l = 1$ g-modes for a rapidly rotating object with similar properties to those expected of GW Lib, we have only shown that a consistent fit is plausible. It is expected that an actual fit of the observed mode frequencies, along with the observed constraints on spin, T_{eff} and mass will provide a measurement of the mass of the accreted layer of solar abundance material. The presence of additional modes is still possible, and these may be at lower or similar frequency to the lowest frequency already observed, as the next few radial order modes are expected to switch from retrograde to prograde from the observer's point of view.

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